Hass, Weir and Thomas: Section 3.9, Exercise 24.

We are given a conical filter 6 inches high and with base a circle with diameter 6 inches and a cylindrical coffeepot with a 6 inch diameter circle as the base.
Coffee is draining from the conical filter into the cylindrical coffeepot at the rate of 10 in^3/min.

1. How fast is the level in the pot rising when the coffee in the cone is 5 in deep?
2. How fast is the level in the cone falling when the coffee in the cone is 5 in deep?

In our discussion in class we had the setup to solutions for both parts of this problem but hadn't finished up. Here are the details.

- Let L denote the level of coffee in the filter.
- Let P denote the level of coffee in the pot.
- The volume of coffee in the pot is $V_P = \pi (3)^2 P = 9\pi P$
    1. So $\frac{dV_F}{dt} = 9\pi \frac{dP}{dt}$
- The volume of coffee in the filter is $V_F = \frac{1}{3}\pi \left(\frac{L}{2}\right)^2 L = \frac{\pi}{12} L^3$ (we used similar triangles to find this volume – see your notes from class.)
- We are given that $\frac{dV_F}{dt} = -10 \ \text{in}^3/\text{min}$ (a minus sign since the coffee is draining out)
- We deduced that $\frac{dV_F}{dt} = -\frac{dV_P}{dt}$ for all t

1. For the first part of the problem, we are asked to find $\frac{dP}{dt}$ when $L = 5$ which we do be relating P and V_P.
    (a) $V_p = 9\pi P$
    (b) $\frac{dV_P}{dt} = 9\pi \frac{dP}{dt}$
    (c) Note that $-10 = \frac{dV_F}{dt} = -\frac{dV_P}{dt} = -9\pi \frac{dP}{dt}$
    (d) So $\frac{dP}{dt} = \frac{10}{9\pi}$ in/min at every time t including the time when $L = 5$.
2. For the second part of the problem we need to relate L and V_F which we do by using $V_F = \frac{\pi}{12} L^3$
    (a) So $\frac{dV_F}{dt} = \frac{3\pi}{12} L^2 \frac{dL}{dt}$ which gives us
    (b) $-10 = \frac{dV_F}{dt} = \frac{\pi}{4} L^2 \frac{dL}{dt}$ and plugging $L = 5$ into this we get
    (c) $-10 = \frac{\pi}{4}\left(5^2\right) \left.\frac{dL}{dt}\right|_{L=5}$
    (d) So $\left.\frac{dL}{dt}\right|_{L=5} = -\frac{10}{25} \cdot \frac{4}{\pi} = \frac{8}{5}\pi$ in/min